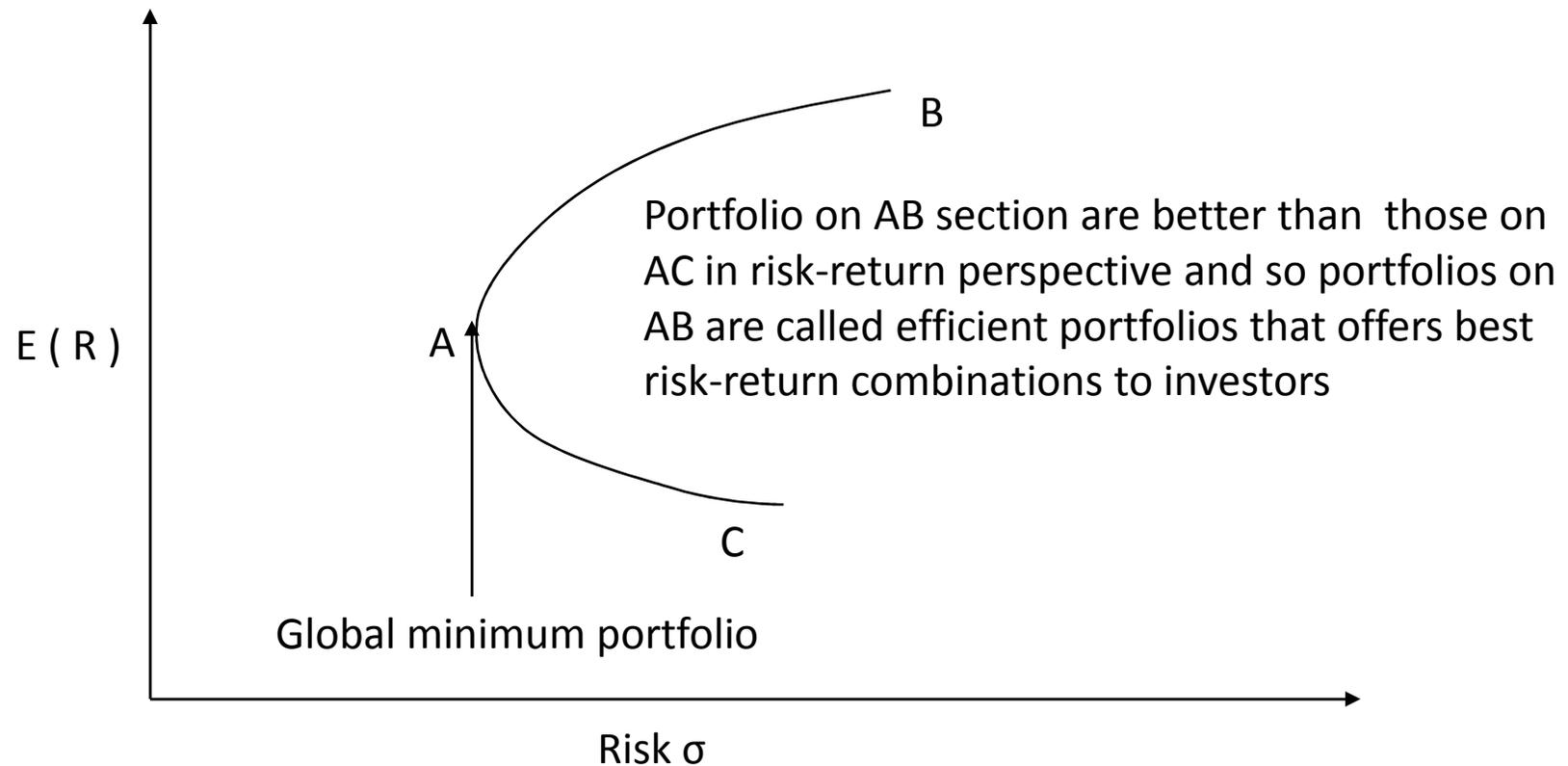


Unit IV: Portfolio Analysis

Single Index Model & Multi Index Model

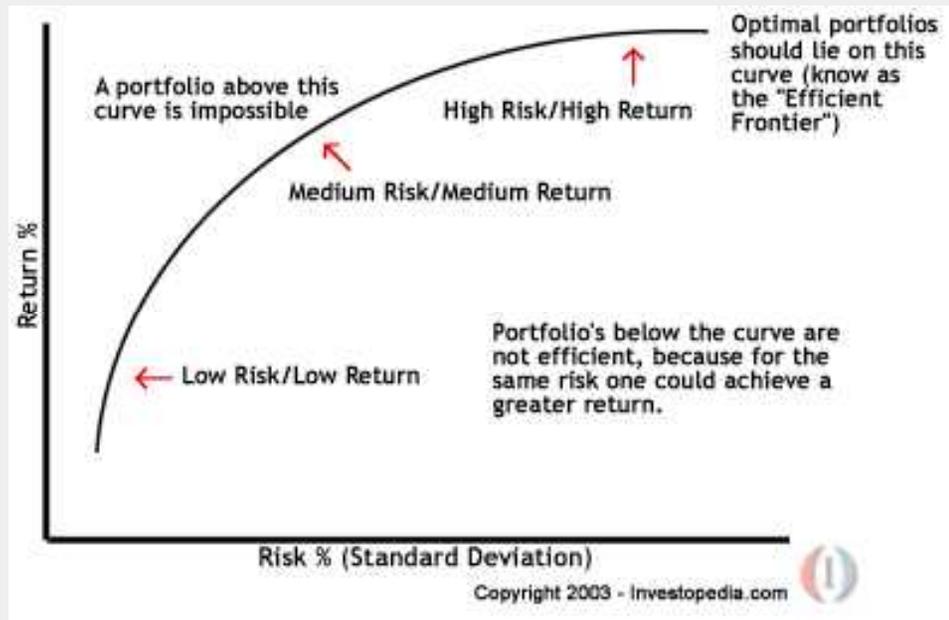
Efficient Frontiers

Graph for the risk-return trade-off according to Markowitz portfolio theory is drawn below.



Computing Problem with Original Markowitz Theory, and Later Simplification

- As n increases, $n(n-1)$ covariances (inputs) are required to calculate under Markowitz model. Due to this complexity of computation, it was mainly used for academic purposes before simplification.
- It was observed that mirror images of covariances were present in Markowitz's model. So after excluding the mirror images in the simplified form, $n(n-1)/2$ unique covariances are required for using this model and since then it is being used by investors.



Markowitz Model for Selection of Optimal Asset Classes-Asset Allocation Decision:

- Markowitz model is typically thought of in terms of selecting portfolios of individual securities. But alternatively, it can be used as a selection technique for asset classes and asset allocation.

Single Index Model - An Alternative Simplified Approach to Determine Efficient Frontiers

- Single-Index model assumes that the risk of return from each security has two components-
 - the market related component($\beta_i R_M$) caused by macro events and
 - the company-specific component(e_i) which is a random residual error caused by micro events.
- The security responds only to market index movement as residual errors of the securities are uncorrelated. The residual errors occur due to deviations from the fitted relationship between security return and market return. For any period, it represents the difference between the actual return(R_i) and the return predicted by the parameters of the model($\beta_i R_M$)

- The Single Index model is given by the equation:

$$R_i = \alpha_i + \beta_i R_M + e_i \dots\dots\dots \text{for security } i, \text{ where}$$

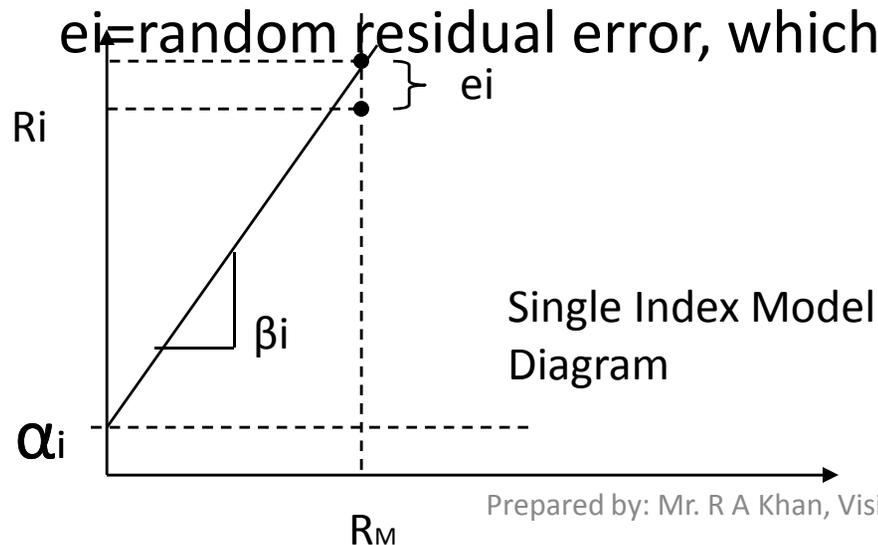
R_i = the return on security

R_M = the return from the market index

α_i = risk free part of security i 's return which is independent of market return

β_i = sensitivity of security i , a measure of change of R_i for per unit change R_M , which is a constant

e_i = random residual error, which is company specific



Single Index Model.....

- Total risk of a security , as measured by its variance, consists of two components: market risk and unique risk and given by

$$\alpha_i^2 = \beta_i^2 [\alpha_M^2] + \alpha_{ei}^2$$

=Market risk + company-specific risk

This simplification also applies to portfolios, providing an alternative expression to use in finding the minimum variance set of portfolios:

$$\alpha_p^2 = \beta_p^2 [\alpha_M^2] + \alpha_{ep}^2$$

- The Single-Index model is an alternative to Markowitz model to determine the efficient frontiers with much fewer calculations, $3n+2$ calculations, instead of $n(n-1)/2$ calculations. For 20 securities, it requires 62 inputs instead of 190 in Markowitz model.

Multi-Index Model

- Some researchers have attempted to capture some non-market influences on stock price by constructing Multi-Index model. Probably the most obvious non-market influence is the industry factor. Multi-index model is given by the equation:

$$E(R_i) = a_i + b_i R_M + c_i NF + e_i, \text{ where } NF = \text{non-market factor}$$

Capital Market Theory(CMT)

- **Capital market theory hypothesizes how investors behave rather than how they should behave as in Markowitz portfolio theory. CMT is based on Markowitz theory and but it is an extension of that.**
- **The more the risk is involved in an investment, the more the return is required to motivate the investors.** It plays a central role in asset pricing, because it is the risk that investors undertake with expectation to be rewarded.
- **CMT is build on Markowitz Portfolio theory and extended with introduction of risk-free asset that allows investors borrowing and lending at risk-free rate and at this, the efficient frontier is completely changed, which in tern leads to a general theory for pricing asset under uncertainty.** Borrowing additional ingestible fund and investing together with investor's wealth allows investors to seek higher expected return, while assuming greater risk. Likewise, lending part of investor's wealth at risk-free rate, investors can reduce risk at the expense of reduced expected return.